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# Spin-fluctuation theory of quasi-two-dimensional itinerant-electron ferromagnets

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**Abstract.** A spin-fluctuation theory of quasi-two-dimensional itinerant-electron ferromagnets is developed. On the basis of an anisotropic spin-fluctuation spectrum, interpolating between two-dimensional (2D) and three-dimensional (3D) cases, we discuss the possibility of observing two-dimensional critical behaviours as well as the crossover phenomena between the 2D and 3D limits.

## 1. Introduction

The magnetism of two-dimensional (2D) itinerant-electron systems has attracted lots of interest since the discovery of cuprate superconductors with layered crystal structures. In the course of the intensive investigations aimed at finding compounds with higher critical temperature, as well as the studies exploring the origin of the superconductivity, a lot of layered compounds were synthesized and studied experimentally. Among them, non-cuprate ruthenate compounds were found to show various interesting properties, such as superconductivity (Maeno *et al* 1994), Mott-insulating properties (Nakatsuji *et al* 1997) and itinerant-electron magnetism (Cao *et al* 1997a, b). Ikeda *et al* (1997) quite recently pointed out the possibility of the appearance of quasi-2D weak ferromagnetism in the $Sr_{3-x}Ca_xRu_2O_7$ system on the basis of magnetic measurements.

In the case of 3D weak itinerant-electron ferromagnets, the self-consistently renormalized (SCR) theory of spin fluctuations (Moriya 1985) has been quite successful. Quantitative comparisons between the theory and experiments have been made, and good agreement has been found. For pure 2D systems, Hatatani and Moriya (1995) developed the spin-fluctuation theory of itinerant ferromagnets by simply extending the SCR theory to 2D cases. They derived various critical behaviours for magnetic and transport properties. For most materials, even for very good layered compounds, there exists a slight three dimensionality. For instance, though actual layered compounds do order ferromagnetically at finite temperature, pure 2D ferromagnets do not have finite critical temperatures. Therefore, for practical comparisons between the theory and experiments for quasi-2D systems, we need to know the conditions and the possibilities for observing 2D critical behaviours as functions of the extent of the three dimensionality.

The purpose of the present paper is therefore to develop the spin-fluctuation theory for quasi-2D itinerant-electron magnets rather than for pure 2D magnets. In the present treatment we deal with 3D itinerant ferromagnets, having anisotropic spin-fluctuation spectra. In this way we will be able to discuss the effects of the slight three dimensionality on

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the possibilities for observing 2D critical behaviours and the crossover phenomena between 2D-like and 3D-like extreme limits.

The present study is also based on the approach proposed by the present author (Takahashi 1986) which is slightly different from the conventional SCR theory. It explicitly takes into account the effects of the zero-point quantum spin fluctuations. From the sum rule for the total spin-fluctuation amplitude I derived various interesting results and consequences (Takahashi 1986, 1990, 1992, 1994, Takahashi and Sakai 1995). According to the formalism, it is shown that the number of independent parameters describing the spin-fluctuation spectrum are reduced, the consequences of which are supported by later experimental investigations (Yoshimura *et al* 1988a, b, Shimizu *et al* 1990, Nakabayashi *et al* 1992). The approach is therefore particularly suited for the mutual comparison of the theory and experiments.

The plan of the paper is as follows. In the next section, we derive the spin-fluctuation spectrum for a free-electron-gas model with an anisotropic dispersion relation. On the basis of the model the form of the anisotropic spin-fluctuation spectrum is derived. Then, with the use of the spectrum, spin-fluctuation amplitudes are evaluated in section 3. In section 4, magnetic properties of itinerant quasi-2D ferromagnets are derived, paying particular attention on the effect of the dimensionality. The final section is devoted to conclusions and discussion.

## 2. Spin-fluctuation spectra for quasi-two-dimensional itinerant ferromagnets

In order to see the effects of quasi-two dimensionality on the spin-fluctuation spectrum of the system, we derive here the form of the non-interacting dynamical magnetic susceptibility based on the free-electron-gas model with different effective masses for the electron motion within the $xy$-plane and along the $z$-axis direction, respectively, i.e. with the following dispersion relation:

$$\varepsilon_{\boldsymbol{k}} = \frac{\hbar^2}{2m}(k_x^2 + k_y^2) + \frac{\hbar^2}{2m'}k_z^2 = \frac{\hbar^2}{2m}(k_x^2 + k_y^2 + \varepsilon^2 k_z^2) \qquad (\varepsilon^2 = m/m'). \tag{1}$$

The effective-mass ratio introduced above as the parameter $\varepsilon^2$ is assumed to be small for quasi-2D systems. By changing the value of $\varepsilon$ from 0 to 1 we can smoothly interpolate between the 2D and 3D limits.

The frequency- and wave-vector-dependent dynamical magnetic susceptibility of the system is evaluated by performing the following wave-vector summation over the wave-vector $\boldsymbol{k}$:

$$\chi_{2\mathrm{d}}^0(\boldsymbol{q}, \omega - \mathrm{i}\delta) = \sum_{\boldsymbol{k}} \frac{f(\varepsilon_{\boldsymbol{k}+\boldsymbol{q}}) - f(\varepsilon_{\boldsymbol{k}})}{\omega - \mathrm{i}\delta + \varepsilon_{\boldsymbol{k}} - \varepsilon_{\boldsymbol{k}+\boldsymbol{q}}} \tag{2}$$

where $f(\varepsilon)$ is the Fermi distribution function. Let us now introduce the new variables $\boldsymbol{K}$ and $\boldsymbol{Q}$ instead of $\boldsymbol{k}$ and $\boldsymbol{q}$ by putting

$$\begin{aligned} &K_x = k_x \qquad K_y = k_y \qquad K_z = \varepsilon k_z \\ &Q_x = q_x \qquad Q_y = q_y \qquad Q_z = \varepsilon q_z. \end{aligned} \tag{3}$$

Then the conduction electron energy dispersion becomes isotropic with respect to the new wave-vector $\boldsymbol{K}$ and the energy difference in the denominator of (2) also has the same form as that of the free-electron-gas model with the isotropic dispersion relation with the effective mass $m$ as shown below:

$$\varepsilon_{\boldsymbol{k}+\boldsymbol{q}} - \varepsilon_{\boldsymbol{k}} = \frac{\hbar^2}{2m}(2k_x q_x + 2k_y q_y + q_x^2 + q_y^2) + \frac{\hbar^2}{2m'}(2k_z q_z + q_z^2)$$

$$= \frac{\hbar^2}{2m}(2\boldsymbol{K}\cdot\boldsymbol{Q} + Q^2). \tag{4}$$

Therefore if we perform the wave-vector summation in (2) in terms of $\boldsymbol{K}$ instead of $\boldsymbol{k}$, we obtain the following relation for the dynamical susceptibilities connecting the isotropic and the anisotropic quasi-2D systems:

$$\chi^0_{2\mathrm{d}}(\boldsymbol{q},\omega) = \frac{1}{\varepsilon}\chi^0_{3\mathrm{d}}(\boldsymbol{Q},\omega)$$

where $\chi^0_{3\mathrm{d}}$ is the dynamical magnetic susceptibility for the 3D non-interacting electron gas system with the isotropic effective mass $m$. The above relation is justified only for an idealized electron gas model without the wave-vector cut-off. In actual cases for electrons in a crystalline lattice, since there is a zone-boundary wave-vector, we have to take the effect of the cut-off into consideration in dealing with the summation with respect to $\boldsymbol{K}$. Then the above relation will not actually hold and some modifications are needed. Nevertheless from the above arguments we could still assume that the $q$-dependence can be represented in terms of the function of $\boldsymbol{Q}$ defined in (3). We therefore expect the following expansion form for $\chi^0_{2\mathrm{d}}(\boldsymbol{q})$, similar to that for $\chi^0_{3\mathrm{d}}$ in the small-wave-vector and low-frequency region:

$$\begin{aligned}\chi^0_{2\mathrm{d}}(\boldsymbol{q},\omega) &= \chi^0_{2\mathrm{d}}(0,0) - AQ^2 + \mathrm{i}C\omega/Q + \cdots \\ &= \chi^0_{2\mathrm{d}}(0,0) - A(q_x^2+q_y^2+\varepsilon^2 q_z^2) + \mathrm{i}C\omega/(q_x^2+q_y^2+\varepsilon^2 q_z^2)^{1/2} + \cdots.\end{aligned} \tag{5}$$

We see that the $q^2$-coefficient of the wave-vector dependence of the static susceptibility is reduced along the $z$-direction by a factor of $\varepsilon^2$, while the $\omega$-linear coefficient is enhanced by the factor $1/\varepsilon$. Although the above expansion form is derived for the free-electron-gas model, it will be extended to realistic situations with complex band structures. Then the coefficients $A$ and $C$ are evaluated on the basis of the details of the band structure of the system.

## 3. Spin fluctuations in quasi-2D systems

Our present discussions, detailed below, are based on the following sum rule for the total spin-fluctuation amplitude (Takahashi 1986), i.e. the total amplitude at each magnetic lattice site:

$$\langle \boldsymbol{S}^2\rangle_{\mathrm{total}} = \langle \boldsymbol{S}^2\rangle_Z + \langle \boldsymbol{S}^2\rangle_T \tag{6}$$

is conserved. The suffices Z and T on the right-hand side stand for the zero-point (quantum) and the thermal components, respectively. In the absence of either an external or an internal magnetic field, the above components are evaluated from

$$\begin{aligned}\langle \boldsymbol{S}^2\rangle_Z &= \frac{3}{N_0^2}\frac{\Omega}{(2\pi)^3}\sum_{\boldsymbol{q}}\int_0^\infty \frac{\mathrm{d}\omega}{\pi}\ \mathrm{Im}\,\chi(q) \\ \langle \boldsymbol{S}^2\rangle_T &= \frac{6}{N_0^2}\frac{\Omega}{(2\pi)^3}\sum_{\boldsymbol{q}}\int_0^\infty \frac{\mathrm{d}\omega}{\pi}\ n(\omega)\,\mathrm{Im}\,\chi(q)\end{aligned} \tag{7}$$

in terms of the imaginary part of the dynamical susceptibility $\mathrm{Im}\,\chi(q,\omega)$, where $n(\omega)$ is the Bose factor defined by $[\exp(\beta\omega)-1]^{-1}$ ($\beta = 1/kT$) and $N_0$ is the number of magnetic ions in the crystal. We have shown in the preceding section that the dynamical susceptibility for the quasi-2D system has the same form as the 3D case upon making substitution (3). In the cases of near ferromagnetic and weakly ferromagnetic systems, because of the mutual intra-atomic Coulomb correlation, the small-$q$, small-$\omega$ region of the dynamical susceptibility is strongly enhanced and its imaginary part $\mathrm{Im}\,\chi(q,\omega)$ is represented in the form

$$\mathrm{Im}\,\chi(q,\omega) = \chi(q)\frac{\omega\Gamma_q}{\omega^2+\Gamma_q^2}$$

where the wave-vector-dependent static magnetic susceptibility $\chi(q)$ and the damping constant $\Gamma_q$ are given in terms of $\boldsymbol{Q}$ in (3) by

$$\chi(q) = \frac{\chi}{1+Q^2/\kappa^2} \qquad \Gamma_q = \Gamma_0 Q(\kappa^2+Q^2). \tag{8}$$

The parameter $\Gamma_0$ and the correlation length $\kappa^{-1}$ are shown to be related to the expansion coefficients in (5) (Moriya 1985). The wave-vector $\boldsymbol{Q}$ will be used in place of $\boldsymbol{q}$ in the following discussions. Following on from our previous investigations, we also introduce two energetic scales $T_0$ and $T_A$ in order to characterize the spectral distribution of the spin-fluctuation spectrum (8) in the frequency and wave-vector spaces, respectively, via

$$kT_0 = \Gamma_0 q_B^3/(2\pi)^3 \qquad kT_A = N_0 q_B^2/(2\chi\kappa^2). \tag{9}$$

The magnetic susceptibility in units of $(g\mu_B)^2$ (with $g=2$) and the temperature are, on the other hand, represented in terms of the dimensionless reciprocal magnetic susceptibility $y$ and the reduced temperature $t$ defined by

$$y = N_0/2kT_A\chi \qquad t = T/T_0. \tag{10}$$

Then $y$ is related to the magnetization $\sigma$ per magnetic ion in units of $\mu_B$ and the external magnetic field $H$ by

$$y = \frac{1}{kT_A}\frac{\sigma}{h} \qquad (h = 2\mu_B H). \tag{11}$$

In subsequent subsections the $t$- and/or $y$-dependence of the spin-fluctuation amplitudes will be derived. With these results, in conjunction with the use of the sum rule (6), we can derive the equation governing the $t$- and $\sigma$-dependences of the reciprocal susceptibility $y$.

### 3.1. *The thermal fluctuation amplitude*

The thermal amplitude has both $t$- and $y$-dependence. Because of the Bose factor, it is particularly sensitive to the low-energy form of the spin-fluctuation spectrum. The integration with respect to $\omega$ in (7) is easily performed, giving

$$\begin{aligned}\langle \boldsymbol{S}^2\rangle_T &= \frac{6}{N_0^2}\sum_{\boldsymbol{q}}\chi(q)\Gamma_q\int_0^\infty \frac{\mathrm{d}\omega}{\pi}\, n(\omega)\frac{\omega}{\omega^2+\Gamma_q^2} \\ &= \frac{3T_0}{N_0T_A}\sum_{\boldsymbol{q}} x[\ln u - 1/2u - \psi(u)] \qquad (u = \beta\Gamma_q/2\pi)\end{aligned} \tag{12}$$

where $\psi(u)$ is the digamma function.

The effect of the dimensionality, which is of most interest in the present study, comes into play from the wave-vector summation in (7). If we transform the summation into the integral form, the integrand is regarded as the function $F(Q)$ which depends only on the magnitude $Q$. Note, however, that in evaluating such an integral over the whole Brillouin zone of $\boldsymbol{q}$, there appears a $Q$-dependent phase volume because of the restriction on the range of the $z$-component $Q_z$. Since

$$Q_z^2 = Q^2\cos^2\theta \leqslant \varepsilon^2 q_B^2$$

the $\boldsymbol{q}$-summation of $F(Q)$ can be represented by

$$\sum_{\boldsymbol{q}} F(Q) = \frac{1}{\varepsilon}\sum_{\boldsymbol{Q}} F(Q) = \frac{1}{\varepsilon}\frac{\Omega}{(2\pi)^3}\int \mathrm{d}^3Q\ F(Q)$$

and

$$\frac{1}{\varepsilon}\int \mathrm{d}^3Q\ F(Q) = \frac{4\pi q_B^3}{\varepsilon}\int_0^{\varepsilon} x^2\ \mathrm{d}x\ F(Q) + 4\pi q_B^3 \int_{\varepsilon}^{1} x\ \mathrm{d}x\ F(Q) \tag{13}$$

where $x$ is the reduced wave-vector, $Q/q_B$, defined with respect to the zone-boundary vector $q_B$ given by the condition

$$\sum_{\boldsymbol{Q}} 1 = \frac{4\pi\Omega q_B^3}{(2\pi)^3}\left\{\frac{1}{2}(1-\varepsilon^2) + \frac{\varepsilon^2}{3}\right\} = N_0. \tag{14}$$

After performing the $\boldsymbol{Q}$-summation according to (13), we see that the thermal spin-fluctuation amplitude can be represented as the sum of two contributions:

$$\begin{aligned}
&\langle \boldsymbol{S}^2\rangle_T = 3d_T\frac{T_0}{T_A}tT(y,t) = 3d_T\frac{T_0}{T_A}t\,\{T_{3\mathrm{d}}(\eta,\tau) + T_{2\mathrm{d}}(\eta,\tau)\}\\
&T_{3\mathrm{d}}(\eta,\tau) = \frac{1}{\tau}\int_0^1 z^3\ \mathrm{d}z\ [\ln v - 1/2v - \psi(v)]\\
&T_{2\mathrm{d}}(\eta,\tau) = \frac{1}{\tau}\int_1^{1/\varepsilon} z^2\ \mathrm{d}z\ [\ln v - 1/2v - \psi(v)]\\
&v = z(\eta + z^2)/\tau \qquad d_T = \frac{6}{3-\varepsilon^2}
\end{aligned} \tag{15}$$

where we have introduced the new integration variable $z$ which stands for $x/\varepsilon$. From the $z$-dependence (and therefore the $q$-dependence) of the phase volume, it is easy to see that $T_{3\mathrm{d}}$ and $T_{2\mathrm{d}}$ represent the amplitudes arising from 3D-like and 2D-like spin fluctuations, respectively. For small $\varepsilon$, both of the quantities depend universally on $y$ and $t$ through $\eta$ and $\tau$ defined by

$$\eta = y/\varepsilon^2 \qquad \tau = t/\varepsilon^3. \tag{16}$$

We show below how their relative importance will change according to the magnitude of the dimensionality parameter $\varepsilon$ at the critical point.

The thermal amplitude at the critical point $t = t_c$ is evaluated by assuming $y = 0$. The 2D contribution $T_{2\mathrm{d}}(0,\tau)$ is explicitly given by

$$T_{2\mathrm{d}}(0,\tau) = \frac{1}{3}\int_{1/\tau}^{1/\varepsilon^3\tau} \mathrm{d}w\left[\ln w - \frac{1}{2w} - \psi(w)\right] = \frac{1}{3}\{G(1/t_c) - G(1/\tau_c)\} \tag{17}$$

where $G(x)$ is the function given in terms of the Gamma function $\Gamma(x)$ by

$$G(x) = (x - 1/2)\ln x - x - \ln\Gamma(x) + \ln\sqrt{2\pi}$$

which behaves as $-1/(12x)$ for large $x$. On the other hand, the 3D contribution $T_{3\mathrm{d}}(0,\tau)$ is represented in the following integral form:

$$T_{3\mathrm{d}}(0,\tau) = \tau^{1/3}\int_0^{1/\tau^{1/3}} z^3\ \mathrm{d}z\left[\ln z^3 - \frac{1}{2z^3} - \psi(z^3)\right]. \tag{18}$$

In the limit of large $\tau_c = t_c/\varepsilon^3$, the following limiting behaviours are obtained:

$$T_{3\mathrm{d}}(0, \tau_c) \simeq \frac{1}{2}$$
$$T_{2\mathrm{d}}(0, \tau_c) \simeq \frac{1}{6}\ln(\tau_c). \tag{19}$$

In this limit the 2D fluctuations dominate over the 3D ones, and the system behaves like a 2D system. In the opposite limit of $\tau_c \ll 1$, on the other hand,

$$T_{3\mathrm{d}}(0, \tau_c) \simeq c\tau_c^{1/3}$$
$$T_{2\mathrm{d}}(0, \tau_c) \simeq \tau_c/36 \tag{20}$$
$$c = 3^{-3/2}(2\pi)^{-1/3}\Gamma(4/3)\zeta(4/3) = 0.3353\ldots$$

hold and the SCR result for the 3D case is recovered. It is interesting to note here that the relative importance of the 2D and the 3D fluctuation amplitudes is governed by the magnitude of the single parameter $\tau_c$. In order to see the effect of the dimensionality for general $\tau_c$-values, we show in figure 1 the results of numerical calculations for the amplitudes at the critical point $t_c$. We see from the figure that the 2D and 3D contributions are comparable even around the large $\tau_c$-value of about $10^2$.

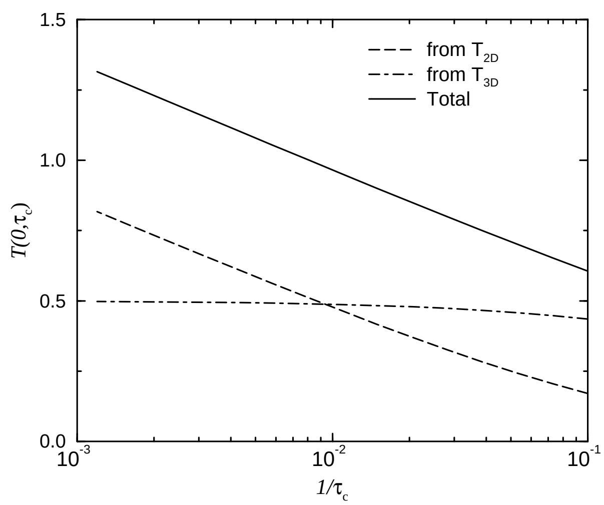


**Figure 1.** The thermal spin-fluctuation amplitude at $t = t_c$ as a function of $1/\tau_c$. The solid, dashed and chain curves represent the total, 2D and 3D fluctuation amplitudes, respectively.

### 3.2. *The quantum fluctuation amplitude*

As was pointed out by Takahashi (1986), though the quantum spin-fluctuation amplitude defined in (7) does not have an explicit temperature dependence, we need to take it account through that of the magnetic susceptibility or its dependence on $y$, whose derivation is the object of the present subsection. In the same way as that of the thermal amplitude, the frequency integration of the quantum amplitude in (7) can be performed as follows:

$$\langle \boldsymbol{S}^2\rangle_Z(y) = \frac{3}{N_0^2}\sum_{\boldsymbol{q}}\chi(q)\Gamma_q\int_0^{\omega_c}\frac{\mathrm{d}\omega}{\pi}\,\frac{\omega}{\omega^2+\Gamma_q^2}$$

$$= \frac{3T_0}{2N_0T_A}\sum_q x\{\ln[\tilde{\omega}_c^2 + x^2(y+x^2)^2] - 2\ln[x(y+x^2)]\}$$
$$= \langle S^2\rangle_Z(0) - \frac{18T_0}{(3-\varepsilon^2)T_A}\left\{\frac{1}{\varepsilon}\int_0^{\varepsilon} x^3\,\mathrm{d}x + \int_{\varepsilon}^{1} x^2\,\mathrm{d}x\right\}$$
$$\times\left[\ln\left(1+\frac{x^2y(1+2x^2y)}{\tilde{\omega}_c^2+x^6}\right) - 2\ln\left(1+\frac{y}{x^2}\right)\right]. \quad (21)$$

We have introduced above the upper cut-off frequency $\omega_c$ (and $\tilde{\omega}_c = \omega_c/2\pi kT_0$), which may depend on the reduced wave-vector $x$. Since we are dealing with the case with small $y$-values, the above expression can be expanded as follows:

$$\langle S^2\rangle_Z(y) = \langle S^2\rangle_Z(0) - 3d_Z\frac{T_0}{T_A}y + \cdots$$
$$d_Z = \frac{6(1-\varepsilon/2)}{3-\varepsilon^2}. \quad (22)$$

The coefficient $d_Z$ is almost independent of $\omega_c$, while the amplitude $\langle S^2\rangle_Z(0)$ at $t = t_c$ depends on its details. According to the space dimension $d$, the value of $d_Z$ is given by

$$d_Z = \begin{cases} 2 & \text{for } d = 2 \quad (\varepsilon = 0) \\ 3/2 & \text{for } d = 3 \quad (\varepsilon = 1). \end{cases} \quad (23)$$

## 4. Magnetic properties of quasi-2D systems

In the presence of the static uniform magnetization $\sigma$ per magnetic atom, the fluctuation amplitudes become anisotropic, and the total amplitude has to be represented as the sum of all the contributions from the longitudinal fluctuations, transverse fluctuations and the static uniform magnetization. With the use of (15) and (22), the sum rule (6) is written in the following form:

$$\langle S^2\rangle_T(0,t_c) = \langle S^2\rangle_{\text{total}} - \langle\delta S^2\rangle_Z(0) = \frac{\sigma^2}{4} - d_Z\frac{T_0}{T_A}(2y+y_z) + d_T\frac{T_0}{T_A}t\{2T(y,t)+T(y_z,t)\} \quad (24)$$

where $\delta S$ is the spin-deviation operator, $S - \langle S\rangle$, and $y_z$ is the reduced longitudinal reciprocal magnetic susceptibility, $(\partial\sigma/\partial h)^{-1}$, given by

$$y_z = y + \sigma\frac{\mathrm{d}y}{\mathrm{d}\sigma}. \quad (25)$$

In deriving (24) we take into account the presence of the two independent degrees of freedom (in the $x$- and $y$-axis directions, for example) for the transverse spin fluctuations. From the requirement of the rotational invariance in spin space, uniform limits of the reciprocal magnetic susceptibility are assumed to be given by $y$ in (11) and $y_z$ for the transverse and the longitudinal components, respectively. The magnetization process and the temperature dependence of the magnetic susceptibility are discussed in the following on the basis of equation (24).

### *4.1. Magnetization processes*

In the ground state the thermal amplitudes vanish identically in (24). Therefore the magnetization process is obtained by solving the following first-order differential equation for $y$

with respect to $\sigma$:

$$\langle S^2\rangle_T(0,t_c) = \langle S^2\rangle_{\mathrm{total}} - \langle \delta S^2\rangle_Z(0) = \frac{\sigma^2}{4} - d_Z\frac{T_0}{T_A}(2y+y_Z) = \frac{\sigma^2}{4} - d_Z\frac{T_0}{T_A}\left(3y+\sigma\frac{\mathrm{d}y}{\mathrm{d}\sigma}\right). \tag{26}$$

It is easy to find the following solution for $y$:

$$y = -\frac{1}{3d_Z}\frac{T_A}{T_0}\langle S^2\rangle_T(0,t_c) + \frac{1}{20d_Z}\frac{T_A}{T_0}\sigma^2. \tag{27}$$

From the condition $y = 0$ in the absence of an external magnetic field, the saturation moment $\sigma_s$ in the ground state is given in each of the 2D and 3D limits by

$$\frac{\sigma_s^2}{4} = \frac{5}{3}\langle S^2\rangle_T(0,t_c) = 5d_T\frac{T_0}{T_A}t_c T(0,t_c) = \begin{cases} \dfrac{5T_0}{3T_A}t_c(3+\ln\tau_c) & \text{for } \tau_c \gg 1 \\ \dfrac{15T_0}{T_A}ct_c\tau_c^{1/3} & \text{for } \tau_c \ll 1. \end{cases} \tag{28}$$

Since $y$ is related to $\sigma$ and $h$ by (11), we see from (27) that the magnetization process of the system in the ground state is given by

$$h = -\frac{1}{3d_Z}\frac{T_A^2}{T_0}\langle S^2\rangle_T(0,t_c)\sigma + \frac{1}{20d_Z}\frac{T_A^2}{T_0}\sigma^3.$$

Now by comparing the result with the definition of the fourth-order coefficient of the magnetic free energy with respect to $\sigma$, $h = \bar{F}_1\sigma(-\sigma_s^2+\sigma^2)/8$, we obtain

$$\bar{F}_1 = \frac{2}{5d_Z}\frac{T_A^2}{T_0}. \tag{29}$$

The value of $\bar{F}_1$ is related to the slope of the Arrott plot (the $\sigma^2$ versus $h/\sigma$ plot). Depending on the value of $\varepsilon$, the value of $\bar{F}_1$ slightly changes its magnitude from $4kT_A^2/15T_0$ for 3D cases to $kT_A^2/5T_0$ for 2D cases.

At finite temperature, the thermal spin-fluctuation amplitude change its $\eta$-dependence as $\eta$ increases in magnitude as shown below. It follows then that we expect crossover phenomena between 3D and 2D critical behaviours of the magnetization process as we increase the value of $\sigma$. As a typical example, let us discuss below the magnetization process at the critical point $t = t_c$ in the 2D limit $\tau_c \gg 1$.

When $\sigma$ is very small and $\eta \ll 1$ is satisfied, we obtain

$$\begin{aligned} T_{3\mathrm{d}}(\eta,\tau) &\simeq T_{3\mathrm{d}}(0,\tau) - \frac{\pi}{4}\sqrt{\eta} \\ T_{2\mathrm{d}}(\eta,\tau) &\simeq T_{2\mathrm{d}}(0,\tau) - \frac{1}{4}(1-\varepsilon)\eta. \end{aligned} \tag{30}$$

In this limit the 3D fluctuations are dominant and the magnetization process is determined by solving

$$\frac{\sigma^2}{4} = d_T\frac{\pi T_0}{4T_A}t_c(2\sqrt{\eta}+\sqrt{\eta_z}). \tag{31}$$

The solution is given in the form $\eta = \eta_c\sigma^4$ with the coefficient

$$\eta_c = \left[\frac{1}{(2+\sqrt{5})\pi d_T}\frac{T_A}{t_c T_0}\right]^2. \tag{32}$$

With the use of (28) the above solution can also be represented as follows:

$$\eta = \left[\frac{20(3+\ln\tau_c)}{3(2+\sqrt{5})\pi d_T}\right]^2 \left(\frac{\sigma}{\sigma_s}\right)^4. \tag{33}$$

Because of the condition $\eta \ll 1$, the validity of the above expressions is limited to the following region of $\sigma$:

$$\left(\frac{\sigma}{\sigma_s}\right)^2 \ll \frac{3\pi(2+\sqrt{5})d_T}{20(3+\ln\tau_c)}. \tag{34}$$

With increasing $\tau_c$, the dominant 3D critical region of $(\sigma/\sigma_s)$ decreases proportionally to $(\ln\tau_c)^{-1/2}$.

On the other hand, as we increase $\sigma$ and if the condition $\eta \gg 1$ is satisfied, we will go over into the region where the 2D fluctuation is dominant, where the $y$-dependences of the thermal amplitudes are well represented by

$$\begin{aligned} T_{3\mathrm{d}}(\eta,\tau) &\simeq T_{3\mathrm{d}}(0,\tau) - \frac{1}{6\eta} \\ T_{2\mathrm{d}}(\eta,\tau) &\simeq T_{2\mathrm{d}}(0,\tau) - \frac{1}{4}\ln\eta. \end{aligned} \tag{35}$$

The magnetization process in this case has to be determined by solving

$$\frac{T_A}{T_0}\frac{\sigma^2}{4} = d_Z\varepsilon^2(2\eta+\eta_z) + \frac{1}{4}d_T t_c(2\ln\eta + \ln\eta_z). \tag{36}$$

The second logarithmic dependence comes from the effect of the 2D fluctuations. If the second term in the right-hand side dominates over the first one when $\varepsilon^2$ is very small, we are led to the following peculiar magnetization process:

$$\eta \simeq \exp\left(\frac{1}{3d_T t_c}\frac{T_A}{T_0}\sigma^2\right) \tag{37}$$

specific to pure 2D itinerant ferromagnets. As we further increase $\sigma$, $y$ (or $\eta$) will finally show $\sigma^2$-linear dependence, showing good linearity when plotted in the form of an Arrott plot.

On the other hand, when $\varepsilon^2$ is of the same order of magnitude as or larger than $t_c$, the above behaviour (37) is not observed. In that case after the 3D critical behaviour for very small $\sigma$, the first $\eta$-linear ($\eta_z$-linear) term in the right-hand side of (36) soon becomes significant, and $\sigma^2$-linear behaviour of $y$ immediately follows as we increase $\sigma$. We expect that the latter case will be realized for actual situations because of the very low $t_c$-values for most itinerant-electron weak ferromagnets. The only effect observed which is a 2D effect is the narrowing of the 3D critical region of the $\sigma^4$-dependence of $y$ at the critical point.

### *4.2. The temperature dependence of the magnetic susceptibility*

From (24), the temperature dependence of the reciprocal magnetic susceptibility $y$ is evaluated from

$$d_Z y = d_T[tT(y,t) - t_c T(0,t_c)]. \tag{38}$$

Because of the change of the $y$-dependence of the thermal amplitude with increasing $t$, the temperature dependence of the susceptibility also shows crossover behaviour depending on its temperature range. The purpose of the present subsection is to discuss the crossover phenomena of the magnetic susceptibility in the limit of $\tau_c \gg 1$.

Near the critical point very close to $t_c$, if $\eta \ll 1$ is satisfied, the system behaves like a 3D one and the $t$-dependence of $y$ is determined by

$$\varepsilon^2 \frac{d_Z}{d_T}\eta = \left(\frac{2}{3} + \frac{1}{6}\ln\tau_c\right)(t - t_c) - \frac{\pi t_c}{4}\sqrt{\eta}. \tag{39}$$

We thus obtain the following temperature dependence:

$$\eta = \left[\frac{2(4 + \ln\tau_c)}{3\pi}\right]^2 \left(\frac{t}{t_c} - 1\right)^2. \tag{40}$$

Compared to the 3D cases, there appears an enhancement proportional to $(\ln\tau_c)^2$ in the coefficient of the $(t - t_c)^2$ term. As the result, from the condition $\eta \ll 1$, the 3D critical temperature range is reduced inversely proportionally to $\ln\tau_c$:

$$\frac{t - t_c}{t_c} \ll \frac{3\pi}{2(4 + \ln\tau_c)}. \tag{41}$$

As we increase the temperature and if $\eta \geqslant 1$ is satisfied, the 2D fluctuations now become dominant, and the temperature dependence of $y$ has now to be determined from

$$\varepsilon^2 \frac{d_Z}{d_T}\eta = -t_c T(0, t_c) + \frac{t}{4}\ln(\tau^{2/3}/\eta). \tag{42}$$

If the three dimensionality $\varepsilon$ is much smaller than $t_c$ around $\eta \simeq 1$, the $\varepsilon^2\eta$-linear term in the left-hand side is neglected compared to both of the terms in the right-hand side. Then $\eta$ will show the following temperature dependence (Hatatani and Moriya 1995):

$$\eta = \tau^{2/3}\exp\left(-\frac{4t_c T(0, t_c)}{t}\right). \tag{43}$$

The above exponential dependence is limited to the temperature range $t \leqslant 4t_c T(0, t_c)$ around the critical point. Otherwise, because of the presence of the $\eta$-linear term in the left-hand side, the Curie–Weiss- (CW-) like temperature dependence would immediately follow after the 3D critical behaviour as we increase the temperature $t$. Depending on the relative magnitude of $\varepsilon^2$ compared with $t_c$, two different kinds of temperature dependence of $y$ will therefore be expected, as summarized in table 1.

**Table 1.** The $t$-dependence of the reciprocal magnetic susceptibility $y$.

| | Around the critical region | Above the critical region |
|---|---|---|
| $\varepsilon^2 \ll t_c$ | $(t - t_c)^2 \sim \tau^{2/3}\exp[-4t_c T(0, t_c)/t]$ | CW-like |
| $\varepsilon^2 \geqslant t_c$ | $(t - t_c)^2$ | CW-like |

The upper case in table 1 is again not so easy to realize for most itinerant ferromagnets, because of their very low $t_c$-values. For most cases, the $\eta$-linear term due to the effect of the quantum amplitude soon becomes dominant and we cannot observe the peculiar temperature dependence specific to the pure 2D fluctuations. In order to confirm the above $t$-dependence of $y$, we numerically solved (38) for several $\varepsilon$-values. The results are shown in figure 2. We see that all of the curves show good Curie–Weiss-like temperature dependence above $t_c$ except in the critical region around $t_c$. As we decrease the $\varepsilon$-value from 1, the slope of the curve $\mathrm{d}y/\mathrm{d}t$ at first gradually increases. Then, after having a maximum value of about 0.3, it decreases very slowly on further decreasing $\varepsilon$.

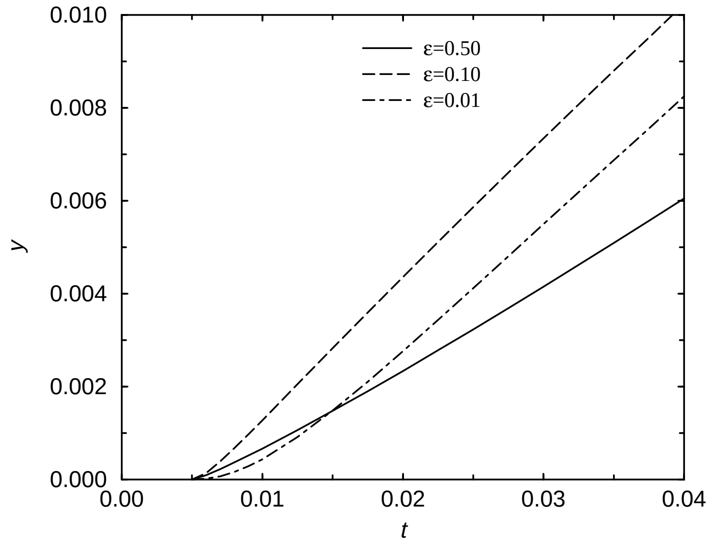


**Figure 2.** The temperature dependence of the reciprocal magnetic susceptibility $y$. The solid curve represents the result for $\varepsilon = 0.5$, the dashed one for $\varepsilon = 0.1$ and the chain one for $\varepsilon = 0.01$.

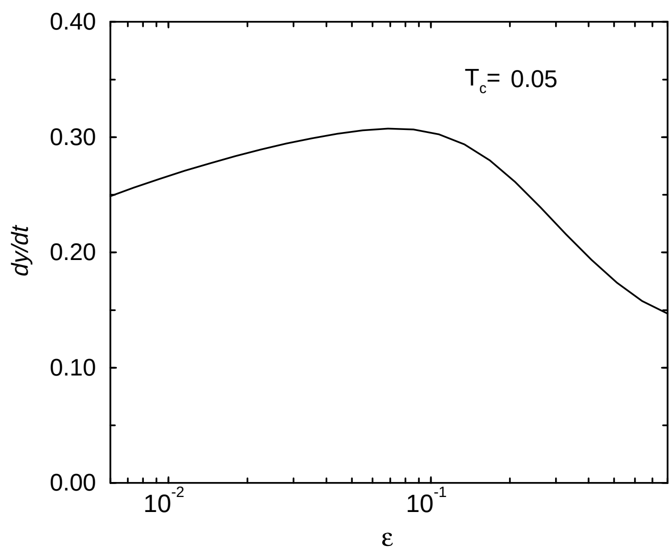


**Figure 3.** The $\varepsilon$-dependence of the derivative $\mathrm{d}y/\mathrm{d}t$ when $t_c = 0.05$ evaluated at $t = 4t_c$.

We show in figure 3 the $\varepsilon$-dependence of the slope $\mathrm{d}y/\mathrm{d}t$. When the magnetic susceptibility shows a temperature dependence according to the Curie–Weiss law, we can define the effective paramagnetic moment $\sigma_{\mathrm{eff}}$ per magnetic ion by

$$\chi = \frac{N_0 \sigma_{\mathrm{eff}}^2}{12k(T - T_c)}$$

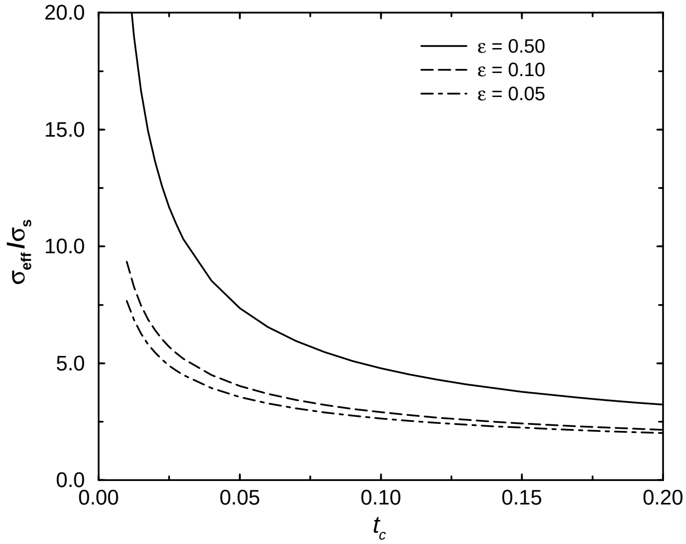


**Figure 4.** The ratio $\sigma_{eff}/\sigma_s$ as a function of $t_c$ for $\varepsilon = 0.5$ (solid curve), for $\varepsilon = 0.1$ (dashed curve) and for $\varepsilon = 0.05$ (chain curve).

in our present units. From the slope of the $t$-linear part of the $y$ versus $t$ curve, we can deduce the Curie constant from

$$\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{6T_0}{T_A\sigma_{eff}^2} = \frac{3}{10d_T t_c T(0,\tau_c)}\left(\frac{\sigma_s}{\sigma_{eff}}\right)^2 \tag{44}$$

where we used (28) in deriving the last line. This means that the ratio $(\sigma_{eff}/\sigma_s)^2$ is related to the slope of $\mathrm{d}y/\mathrm{d}t$, as well as the thermal spin-fluctuation amplitude at $T = T_c$, i.e.

$$\left(\frac{\sigma_{eff}}{\sigma_s}\right)^2 = \frac{3}{10d_T t_c T(0,\tau_c)}\left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^{-1}. \tag{45}$$

In the case of pure 3D cases, the slope $\mathrm{d}y/\mathrm{d}t$ is the universal constant determined by solving (38) with $\varepsilon = 1$ and is independent of any material constants. From (45) we see that the ratio is uniquely determined by the single parameter $t_c$, which serves as the theoretical basis for the revised Rhodes–Wohlfarth plot, i.e. the $\sigma_{\mathrm{eff}}^2/\sigma_{\mathrm{s}}^2$ versus $t_c$ plot (Takahashi 1986). Equation (38) shows that the slopes of $y$ for quasi-2D systems in general depend also on the dimensionality parameter $\varepsilon$.

In order to see the dependence on $\varepsilon$, we show in figure 4 the numerical estimates of $\sigma_{\mathrm{eff}}/\sigma_{\mathrm{s}}$ against $t_c$ for several values of $\varepsilon$. For the same $t_c$-value, smaller values of the ratio $\sigma_{\mathrm{eff}}/\sigma_{\mathrm{s}}$ are obtained with decreasing $\varepsilon$. In the good 2D limit we also see from the figure that the large $\sigma_{\mathrm{eff}}/\sigma_{\mathrm{s}}$ ratio is limited to cases with very small $t_c$-values.

## 5. Discussion

We have developed a spin-fluctuation theory for quasi-two-dimensional itinerant-electron ferromagnets based on a model with an anisotropic spin-fluctuation spectrum. As a result we were able to interpolate between the two extreme 2D and 3D limits in terms of the anisotropy parameter $\varepsilon$. Changing the value of $\varepsilon$, we have discussed the possibility of observing the 2D critical behaviours as well as the crossover phenomena between 2D and 3D behaviours.

We have shown that at the critical point the relative importance of the thermal and the quantum fluctuation amplitudes is governed universally by the single parameter $\tau_c = t_c/\varepsilon^3 = T_c/\varepsilon^3 T_0$. If $\tau_c \gg 1$, 2D fluctuations dominate, while the 3D fluctuation amplitude is dominant when $\tau_c \ll 1$. We have also discussed the crossover phenomena between 2D and 3D critical behaviours. It was also shown that the condition for observing pure 2D-like critical behaviours is very severe, especially for itinerant weak ferromagnets. The main reason for this comes from the low $t_c$-values characteristic of weak itinerant ferromagnets. We conclude therefore that quasi-2D itinerant weak ferromagnets have the following magnetic properties.

(i) $\sigma_s^2$ is proportional to $t_c$ or $t_c \ln \tau_c$ depending on the value of $\tau_c$, rather than on $t_c^{4/3}$ as for 3D cases.

(ii) The formula for the slope of the Arrott plot (29), which relates its slope to the spin-fluctuation spectrum, depends weakly on the dimensionality parameter $\varepsilon$.

(iii) The 3D critical region decreases with increasing two dimensionality of the system.

(iv) For the same value of $t_c$, smaller values of the ratio $\sigma_{\mathrm{eff}}/\sigma_{\mathrm{s}}$ are obtained (see figure 4). Compared to 3D case, the large $\sigma_{\mathrm{eff}}/\sigma_{\mathrm{s}}$ ratio is limited to a narrower region with small $t_c$-values.

The critical behaviours characteristic of pure 2D systems will only be observed when the two dimensionality is very good and the condition $\varepsilon^2 \ll t_c$ is satisfied. In this sense, the recent experiments on the layered quasi-2D ferromagnets by Ikeda *et al*, where no definite 2D critical behaviours have been observed, may be reasonable. The nearly complete absence of critical magnetization behaviour may also be consistent with our conclusions.

Note that our present conclusion on the difficulty of observing pure 2D critical behaviours is specific to itinerant-electron weak ferromagnets, for which only the fluctuations in a small portion of the $q$-space are responsible for the thermal spin-fluctuation amplitude. Because these fluctuations with small $q$-values have long-range spatial correlations and if they are greater than the interlayer distance of layered compounds, for instance, the systems will behave as 3D magnets rather than 2D magnets. Our present study shows that quasi-2D itinerant weak magnets will behave qualitatively like 3D magnets. Nevertheless, to make quantitative comparisons between theory and experiments, we have to take into account various renormalization effects of parameters due to the effects of the dimensionality parameter $\varepsilon$. Analyses have therefore to be done on the basis of the theory taking into account the effects of the 3D and 2D spin fluctuations simultaneously.

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